OCR
RECOGNISING ACHIEVEMENT

**ADVANCED GCE UNIT** **4756/01**

**MATHEMATICS (MEI)**

Further Methods for Advanced Mathematics (FP2)

**TUESDAY 16 JANUARY 2007**

Morning
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions in Section A and **one** question from Section B.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

**ADVICE TO CANDIDATES**

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.



**[Turn over**

**Section A** (54 marks)

**Answer all the questions**

**1** **(a)** A curve has polar equation $r = ae^{-k\theta}$ for $0 \leqslant \theta \leqslant \pi$, where $a$ and $k$ are positive constants. The points A and B on the curve correspond to $\theta = 0$ and $\theta = \pi$ respectively.

**(i)** Sketch the curve. [2]

**(ii)** Find the area of the region enclosed by the curve and the line AB. [4]

**(b)** Find the exact value of $\int_0^{\frac{1}{2}} \frac{1}{3+4x^2}\,dx$. [5]

**(c)** **(i)** Find the Maclaurin series for $\tan x$, up to the term in $x^3$. [4]

**(ii)** Use this Maclaurin series to show that, when $h$ is small, $\int_h^{4h} \frac{\tan x}{x}\,dx \approx 3h + 7h^3$. [3]

**2** **(a)** You are given the complex numbers $w = 3e^{-\frac{1}{12}\pi j}$ and $z = 1 - \sqrt{3}\,j$.

**(i)** Find the modulus and argument of each of the complex numbers $w$, $z$ and $\frac{w}{z}$. [5]

**(ii)** Hence write $\frac{w}{z}$ in the form $a + bj$, giving the exact values of $a$ and $b$. [2]

**(b)** In this part of the question, $n$ is a positive integer and $\theta$ is a real number with $0 < \theta < \frac{\pi}{n}$.

**(i)** Express $e^{-\frac{1}{2}j\theta} + e^{\frac{1}{2}j\theta}$ in simplified trigonometric form, and hence, or otherwise, show that

$$1 + e^{j\theta} = 2e^{\frac{1}{2}j\theta}\cos\tfrac{1}{2}\theta.$$

[4]

Series $C$ and $S$ are defined by

$$C = 1 + \binom{n}{1}\cos\theta + \binom{n}{2}\cos 2\theta + \binom{n}{3}\cos 3\theta + \ldots + \binom{n}{n}\cos n\theta,$$

$$S = \binom{n}{1}\sin\theta + \binom{n}{2}\sin 2\theta + \binom{n}{3}\sin 3\theta + \ldots + \binom{n}{n}\sin n\theta.$$

**(ii)** Find $C$ and $S$, and show that $\frac{S}{C} = \tan\frac{1}{2}n\theta$. [7]

**3** Let $\mathbf{P}=\begin{pmatrix}4 & 2 & k\\ 1 & 1 & 3\\ 1 & 0 & -1\end{pmatrix}$ (where $k \neq 4$) and $\mathbf{M}=\begin{pmatrix}2 & -2 & -6\\ -1 & 3 & 1\\ 1 & -2 & -2\end{pmatrix}$.

**(i)** Find $\mathbf{P}^{-1}$ in terms of $k$, and show that, when $k = 2$, $\mathbf{P}^{-1}=\frac{1}{2}\begin{pmatrix}-1 & 2 & 4\\ 4 & -6 & -10\\ -1 & 2 & 2\end{pmatrix}$. [6]

**(ii)** Verify that $\begin{pmatrix}4\\1\\1\end{pmatrix}$, $\begin{pmatrix}2\\1\\0\end{pmatrix}$ and $\begin{pmatrix}2\\3\\-1\end{pmatrix}$ are eigenvectors of $\mathbf{M}$, and find the corresponding eigenvalues.

[4]

**(iii)** Show that $\mathbf{M}^n=\begin{pmatrix}4 & -6 & -10\\ 2 & -3 & -5\\ 0 & 0 & 0\end{pmatrix}+2^{n-1}\begin{pmatrix}-2 & 4 & 4\\ -3 & 6 & 6\\ 1 & -2 & -2\end{pmatrix}$. [8]

## **Section B** (18 marks)

### Answer one question

*Option 1: Hyperbolic functions*

**4** **(i)** Show that $\operatorname{arcosh} x = \ln\left(x+\sqrt{x^2-1}\right)$. [5]

**(ii)** Find $\displaystyle\int_{2.5}^{3.9} \frac{1}{\sqrt{4x^2-9}}\,dx$, giving your answer in the form $a\ln b$, where $a$ and $b$ are rational numbers. [5]

**(iii)** There are two points on the curve $y = \dfrac{\cosh x}{2+\sinh x}$ at which the gradient is $\frac{1}{9}$.

Show that one of these points is $\left(\ln\left(1+\sqrt{2}\right), \frac{1}{3}\sqrt{2}\right)$, and find the coordinates of the other point, in a similar form. [8]

*Option 2: Investigation of curves*

**This question requires the use of a graphical calculator.**

**5** Cartesian coordinates $(x, y)$ and polar coordinates $(r, \theta)$ are set up in the usual way, with the pole at the origin and the initial line along the positive $x$-axis, so that $x = r\cos\theta$ and $y = r\sin\theta$.

A curve has polar equation $r = k + \cos\theta$, where $k$ is a constant with $k \geqslant 1$.

**(i)** Use your graphical calculator to obtain sketches of the curve in the three cases

$$k = 1, k = 1.5 \text{ and } k = 4.$$ [5]

**(ii)** Name the special feature which the curve has when $k = 1$. [1]

**(iii)** For each of the three cases, state the number of points on the curve at which the tangent is parallel to the $y$-axis. [2]

**(iv)** Express $x$ in terms of $k$ and $\theta$, and find $\frac{dx}{d\theta}$. Hence find the range of values of $k$ for which there are just two points on the curve where the tangent is parallel to the $y$-axis. [4]

The distance between the point $(r, \theta)$ on the curve and the point $(1, 0)$ on the $x$-axis is $d$.

**(v)** Use the cosine rule to express $d^2$ in terms of $k$ and $\theta$, and deduce that $k^2 \leqslant d^2 \leqslant k^2 + 1$. [4]

**(vi)** Hence show that, when $k$ is large, the shape of the curve is very nearly circular. [2]

 